HUB-EP-97/13
hep-th/9702179

February 25, 1997

More on $N = 1$ Self-Dualities and Exceptional Gauge Groups

Andreas Karch *

Humboldt-Universität zu Berlin
Institut für Physik D-10115 Berlin,
Germany

Email:
karch@qft1.physik.hu-berlin.de

Abstract

Starting from a generalization of a recent result on self-duality [15] we systematically analyze self-dual models. We find a criterion to judge whether a given model is self-dual or not. With this tool we construct some new self-dual pairs, focussing on examples with exceptional gauge groups.

*Work supported by DFG

1. Introduction

In the last few years it has become clear that many $N = 1$ supersymmetric gauge theories in four dimensions exhibit the phenomenon of duality ([1], [2]-[15]). Within these huge amount of known examples there is a class of models that deserves special attention: the self-dual ones. Self-duality in this context means that the magnetic gauge group is the same as the electric one, but they do not describe the same UV physics, since the magnetic theory has additional gauge singlets and superpotential terms.

These self-dual models have some very interesting features. One on hand they look very simple. The 't Hooft anomaly matching conditions are always satisfied in a very trivial fashion. They are very helpful in building duals with matter in a tensor representation of the gauge group and without an electric tree level superpotential [13] and they are almost the only information we got so far about exceptional gauge groups other than G_2 ([8,14]). On the other hand they were shown to be related to the existence of exactly marginal operators [11] and this might make them somewhat easier to study.

Recently several new self-dualities have been found in [15] for *SU* groups with various higher rank antisymmetric tensors and for some *Spin* groups with spinors. In this paper we will generalize their *Spin* self-dualities, find a tool to construct new self-dual models and finally use it to get some new information about exceptional gauge groups.

Section 2 contains a review of the *Spin* models of [15] and a series of similar dualities. We will find the first indications that there should be a criterion for possible self-duality involving the object $\Delta = \mu_{gauge} - \mu_{matter}$ like the one found in [16] for the construction of s-confining theories.

In Section 3 we will construct this tool and show why it works and what its limitations are. In Section 4 we then use this new tool to get information about exceptional gauge groups and to construct other new self-dual models.

We will conclude in section 5.

2. The *Spin* series of [15]

In [15] the authors discovered several new selfdual models. Within those was a series of $Spin(N)$ models with $N - 4$ vectors. The important information about those models is contained in the following table, taken from [15].

group	content	“mesons”
$Spin(4)$	$(8,8,0)$	$\sim SU(2)\times SU(2)$ with $8(\Box,1)+8(1,\Box)$
$Spin(5)$	$(8,1)$	$\sim Sp(4)$ with $8\,\Box+\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$
$Spin(6)$	$(4,4,2)$	$\sim SU(4)$ with $4(\Box+\overline{\Box})+2\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$
$Spin(7)$	$(4,3)$	$s^2, s^2q, s^2q^2, s^2q^3$
$Spin(8)$	$(4,0,4)$	s^2, s^2q^2, s^2q^4
$Spin(8)$	$(2,2,4)$	$s^2, s^2q^2, s^2q^4, c^2, c^2q^2, c^2q^4, scq, scq^3$
$Spin(8)$	$(3,1,4)$	$s^2, s^2q^2, s^2q^4, c^2, c^2q^4, scq, scq^3$
$Spin(9)$	$(2,5)$	$s^2, s^2q, s^2q^2, s^2q^3, s^2q^4, s^2q^5$
$Spin(10)$	$(2,0,6)$	s^2q, s^2q^3, s^2q^5
$Spin(10)$	$(1,1,6)$	$s^2q, s^2q^5, c^2q, c^2q^5, sc, scq^2, scq^4, scq^6$
$Spin(11)$	$(1,7)$	$s^2q, s^2q^2, s^2q^5, s^2q^6$
$Spin(12)$	$(1,0,8)$	s^2q^2, s^2q^6

Table 1: Selfdual models of [15]

The first column denotes the electric gauge group, the second column denotes the matter content, (number of spinors, number of vectors) for odd N and (number of spinors, number of conjugate spinors, number of vectors) for even N. Since by definition of what we mean by self-duality, the magnetic gauge group is the same as the electric one, to fully specify the model, it is enough to list the operators becoming fundamental gauge singlets on the magnetic side and whose corresponding magnetic operator gets removed via the superpotential. This information is contained in the third column. [1]

The superpotential is constructed in the obvious way.

For example, the last line contains the data of the following dual pair:

electric:

	$Spin(12)$	$SU(8)$	$U(1)$	$U(1)_R$
Q	12	8	1	$\frac{1}{6}$
S	32	1	-2	$\frac{1}{6}$

magnetic:

	$Spin(12)$	$SU(8)$	$U(1)$	$U(1)_R$
q	12	8	1	$\frac{1}{6}$
s	32	1	-2	$\frac{1}{6}$
M_4	1	$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$	-2	$\frac{2}{3}$
M_8	1	$\overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}}$	2	$\frac{4}{3}$

with: $W = M_4 s^2 q^6 + M_8 s^2 q^2$.

[1]Here and in the following we denote magnetic fields by small letters and electric fields by capital letters. We use q, Q as a name for fields in the fundamental representation of the gauge group (the vector in the *Spin* cases), s, S for spinors and c, C for conjugate spinors.

The gauge invariant operator S^2Q^6 gets mapped to M_8, S^2Q^2 to M_4. s^2q^6 and s^2q^2 are removed via the equations of motion for M_4 and M_8. All other independent gauge invariant operators are the same on both sides and get mapped into each other.

As indicated, the first three entries in Table 1 are equivalent to other theories which were allready known to be self-dual. This is another check for the consistency of the models. Giving a vev to one of the vectors, one can flow from a $Spin(N)$ dual in the list to a $Spin(N-1)$ one. Giving a mass to one of the vectors, one flows to a s-confining model ([16]). All the s-confining *Spin* theories have been classified by the same authors in [16]. The fact, that the list of those s-confining theories contains another similar series of s-confining theories hints that there is also another series of self-dual *Spin* models. In fact it is easy to check, that the following models are also self-dual:

group	content	"mesons"
$Spin(8)$	$(4,4,0)$	s^2, s^2c^2, s^2c^4
$Spin(9)$	$(4,1)$	
$Spin(10)$	$(2,2,2)$	sc^3q, s^3cq
$Spin(11)$	$(2,3)$	$s^2q, s^2q^2, s^6q, s^6q^2$
$Spin(12)$	$(2,0,4)$	s^2q^2, s^6q^2
$Spin(12)$	$(1,1,4)$	$s^2q^2, c^2q^2, s^4c^2q^2, s^2c^4q^2$
$Spin(13)$	$(1,5)$	$s^2q^2, s^2q^3, s^6q^2, s^6q^3$
$Spin(14)$	$(1,0,6)$	s^2q^3, s^6q^3

As evidence for those dualities we take as usual, that the 't Hooft matching conditions for the global symmetries are satisfied and that we can consistently perturb the dual pairs along flat directions to another consistent dual pair. We only checked the vector flat directions, which connect the theories listed in the table with each other. But there is no indication that the spinors should make trouble. The final $Spin(8)$ theory with $(4,4,0)$ is equivalent to the $Spin(8)$ theory with $(4,0,4)$ of [15] by triality.

Mass terms again perturb to s-confining theories, and these two series together now reproduce allmost all the known s-confining models. This close connection between s-confining models and self-dual models is kind of surprising. In the next section we will argue that this is due to the fact that both phenomena can be analyzed by looking at the expression $\mu_{matter} - \mu_{gauge}$. Here μ stands for the quadratic index of the gauge group. μ_{matter} is the sum of all the indices of all the matter fields in the various representations and μ_{gauge} is the index of the gauge fields (the index of the adjoint). We normalize μ to be 1 for a fundamental *SU* representation.

3. A Tool for constructing self-dual models

The quantity $\Delta \equiv \mu_{matter} - \mu_{gauge}$ has already proved to be useful in several cases. [16] showed that $\Delta = 2$ indicates a s-confining theory. Even though this alone was neither necessary nor sufficient, it proved to be very useful for their work. They were able to classify all the s-confining models with little more work than ckecking the above condition: together with their second condition obtained by studying different regions on the moduli space the index

constraint became necessary. This way they created a list of all possible candidate theories and had to check them by their second criterion (going along flat directions). Similary, $\Delta = 0$ indicates a quantum smoothed out moduli space and $\Delta = -2$ a non-perturbative superpotential created by a single instanton effect.

We would like to argue that in a similar way $\Delta = k$ and k a divisor of μ_{gauge} is an indication for self-duality. This is again neither a necessary nor a sufficient condition. Nevertheless it proved to be a useful tool, since it allowed us to construct several new self-dual models, which we will present in the next section.

Before we explain why the above condition is important let us make some comments. Of all the self-dual models without a tree level superpotential we know of ([1,11,8,13,14,15]) only the odd $Spin(N)$ models from above, the $Sp(2n)$ theories with odd n of [13] and some of the models from the next section do not meet the requirement, that Δ is a divisor of μ_{gauge}. And all those models can be obtained by giving a vev to some field in a theory that does meet the requirement [2]. Probably there are other self-dual models not related at all to our requirement. But as will become clear soon, their structure is much more complicated and so no one has guessed them so far. We will have to wait for a better understanding of the mechanisms of duality in $N = 1$ theories before we can try to systematically analyze them.

The case $k = 2$ deserves special attention. There are many theories meeting this requirement. Luckily the $k = 2$ case is exactly the case discussed in [16] that also indicates s-confinement. So most of the work in this case has been done. All the $Spin(N)$ theories from the last section have $\Delta = 4$. For even N this is a divisor of $\mu_{gauge} = 4N - 4$. And the odd ones can be obtained from the even ones by higgsing a vector. Removing a vector by giving it a mass reduces Δ to 2 and gives an s-confining model.

Now let us explain why the requirement $\Delta = k$ and k a divisor of μ_{gauge} is important for self-duality. If one assigns the same r-charge r to all the matter fields, for an anomaly free r-symmetry this has to be $r = 1 - \frac{\mu_{gauge}}{\mu_{matter}}$. The above requirement then translates to the requirement $r = \frac{1}{n}$ for some integer n ($n = \frac{\mu_{gauge}}{k} + 1$). This is important for self-duality for two reasons.

The first is a very technical point of view. To satisfy the 't Hooft matching conditions for the global r-symmetry with the magnetic and the electric gauge group being the same is almost impossible unless the additional gauge singlets on the magnetic side have either r-charge 1 (which means that they do not contribute at all) or always come in pairs of r-charge $1 + a$ and $1 - a$ (in which case their contribution cancels). If the r-charge of all fields is just $\frac{1}{n}$ such operators are very easy to construct: a composite built out of n fields has r-charge 1 and composites built out of $n+m$ fields and $n-m$ fields (m integer and less than n) form an $(1+a, 1-a)$ pair. Even though it is in principal not impossible to construct such operators in a different way or to satisfy the r-anomaly with a different mechanism, this is the easiest

[2] To see this in the Sp(2n) theory of [13] consider giving a vev to the asym. tensor A of the form $< A >= i\sigma_2 v \otimes \begin{pmatrix} 1 & & & \\ & 0 & & \\ & & \ddots & \\ & & & 0 \end{pmatrix}$. This breaks the gauge group to $Sp(2(n-1)) \times SU(2)$. Since the operators involving only A are mapped onto themselves on the magnetic side, exactly the same happens to the magnetic theory. The $SU(2)$ decouples and one obtains the self-dual $n-1$ theory from the n theory.

way to do it and explains, why all the models found so far are either constructed this way or derived from a model constructed this way.

It should allready be clear at this point, why we have trouble with the theories with tree level superpotential: in the presence of a tree level superpotential term on the electric side we loose the freedom to freely choose the r-charge of the fields to be equal. We have to fix it to whatever is preserved by the superpotential.

The second argument uses the deep connection between self-dual models and exactly marginal operators. Following the lines of [11] it is natural to conjecture that in every self-dual model there is an exactly marginal operator that parametrizes a fixed line smoothly connecting the electric and the magnetic theory. The way this marginal operator arises is the following: Consider the magentic theory and add a mass term for the mesons (that is what in the case for self-dual SUSY QCD was called Supersymmetric Quantum Chromesodynamics in [11]) The $m = 0$ limit clearly reproduces the magnetic theory, $m \to \infty$ gives back the electric theory. Integrating out the meson at intermediate values of m generates an operator in the superpotential which is exactly marginal (if there are different types of mesons, a linear combination of the operators created this way is conjectured to be marginal). This structure arose in their case from the close connection to $N = 2$, but the conjecture is supposed to be true even for models with no $N = 2$ origin. Hence we can find self-dual models by searching for theories with exactly marginal operators. We will show that with the r-charge assignment and meson content introduced above (which was only possible if the Δ constrained was satisfied) the model obtained this way has indeed a fixed line instead of just a fixed point and hence an exactly marginal operator.

To establish this, it is necessary to study the β-functions of the gauge coupling and the coupling of a linear combination of the operators associated with the various mesons. These operators are of the form: "gauge invariant operator that becomes a elementary magnetic gauge singlet" times "the gauge invariant operator it removes on the magnetic side via its equation of motion". For example in SUSY QCD we are looking for $(Q\tilde{Q})^2$ (see [11]) and in the $Spin(12)$ theory of [15] discussed in section 2 for $(S^2V^2)(S^2V^6)$, all the indices being contracted.

Since these operators appear in the superpotential they have to have r-charge 2 and hence also any linear combination of them. They all have $2n$ fields (since each field has r-charge $\frac{1}{n}$), and invariance of the superpotential under the various anomaly free $U(1)$s tells us that the number $\#_i$ of fields of type i in anyone of the products in the linear combination that makes up our operator is $\#_i = \frac{2\mu_i}{\Delta}$.

Existence of a fixed line is equivalent to the fact that the scaling coefficents of the gauge coupling A_g and for the superpotential coupling of the operator A_h are linearly dependend.

They are

$$
\begin{aligned}
A_g &= -3\mu_{gauge} + \mu_{matter} - \sum_i \mu_i \gamma_i \\
&= \mu_{gauge} \cdot \frac{3-2n}{n-1} - \sum_i \mu_i \gamma_i \\
A_h &= 2n - 3 + \frac{1}{2} \sum_i \#_i \gamma_i \\
&= -(3-2n) + \sum_i \mu_i \frac{n-1}{\mu_{gauge}} \gamma_i
\end{aligned}
$$

where γ_i denotes the anomalous mass dimension of field type i and we used $\Delta = \frac{\mu_{gauge}}{n-1}$ various times. These are now obviously linearly dependend, since $A_g = -\frac{\mu_{gauge}}{n-1} A_h$.

4. New Self-Dual Models

4.1. Exceptional Gauge Groups

One of the big unsolved problems in $N=1$ dualities is finding a dual description for models with exceptional gauge groups. All the progress made so far is based on self-dual models ([8], [14]). With the tool described in the last section, we were able to construct some new exceptional self-dualities. In the short notation introduced in section 2 they are:

group	content	"mesons"
E_6	$(3,3)$	$q\tilde{q}, q_a^2 \tilde{q}_a^2$
F_4	(4)	$q_s^2, q^6(\boxplus)$
G_2	(6)	q_a^3

With the following Δ values

$$
\begin{aligned}
\Delta(E_6) &= 6\mu_{27} - \mu_{78} = 6 \cdot 6 - 24 = \frac{1}{2}\mu_{78} \\
\Delta(F_4) &= 4\mu_{26} - \mu_{52} = 4 \cdot 6 - 18 = \frac{1}{3}\mu_{52} \\
\Delta(G_2) &= 6\mu_{7} - \mu_{14} = 6 \cdot 2 - 8 = \frac{1}{2}\mu_{14}
\end{aligned}
$$

the three clearly qualify for self-duality by the arguments of the previous section. There is a one to one mapping of gauge invariant operators and the 't Hooft anomaly matchings are satisfied. As further evidence we checked some of the flat directions:

Consider perturbing the F_4 model along the flat direction described by

$$M^{111} = d^{\alpha\beta\gamma} Q^1_\alpha Q^2_\beta Q^3_\gamma,$$

α, β, γ being color indices, 1 a flavor index and d the 3-index totally symmetric invariant tensor of F_4. That is, we give one flavor a vev in such a way, that only the invariant built with d gets a non-zero vev. This higgses the electric gauge theory to $Spin(8)$ with $(3,3,3)$. Since the invariant is mapped to the magnetic one built in exactly the same way, the magnetic gauge group is also $Spin(8)$ with $(3,3,3)$. This is indeed again a new consistent self-dual pair with

$$\Delta(Spin(8)) \quad = \quad 9\mu_8 - \mu_{28} = 18 - 12 = \frac{1}{2}\mu_{28}.$$

and the following content:

	$Spin(8)$	$SU(3)_V$	$SU(3)_S$	$SU(3)_C$	$U(1)$	$U(1)$	$U(1)_R$
Q	8_V	3	1	1	1	1	$\frac{1}{3}$
S	8_S	1	3	1	-1	0	$\frac{1}{3}$
C	8_C	1	1	3	0	-1	$\frac{1}{3}$

$$\updownarrow$$

	$Spin(8)$	$SU(3)_D$	$U(1)$	$U(1)$	$U(1)_R$
q	8_V	3	1	1	$\frac{1}{3}$
s	8_S	3	-1	0	$\frac{1}{3}$
c	8_C	3	0	-1	$\frac{1}{3}$
M_2	1		2	2	$\frac{2}{3}$
N_2	1	$\square\!\square$	-2	0	$\frac{2}{3}$
P_2	1		0	-2	$\frac{2}{3}$
M_4	1		-2	-2	$\frac{4}{3}$
N_4	1	$\boxplus$	$+2$	0	$\frac{4}{3}$
P_4	1		0	$+2$	$\frac{4}{3}$

with: $W = M_2 c_a^2 s_a^2 + N_2 q_a^2 c_a^2 + P_2 q_a^2 s_a^2 + M_4 q_s^2 + N_4 s_s^2 + P_4 c_s^2$.

On the magnetic side only the diagonal subgroup of the electric $SU(3) \times SU(3) \times SU(3)$ global flavor rotations is visible on the fundamental magnetic fields. The full symmetry only appears as an accidental symmetry in the far IR. This is by now a well known phenomenon, that appeared over and over again ([10,14,15]). It is not surprising to see it pop up here: the diagonal $SU(3)$ is the one inherited from the F_4 theory. On the electric side the only objects charged under this $SU(3)$ are the 27s, which split up into three objects under the broken gauge group, which can be rotated independently. On the magnetic side there are also the mesons transforming under the global flavor rotations. They are not affected by the higgsing and there is no reason why they should transform under three seperate rotations. Their superpotential contribution prevents one from rotating the three different 8s independently. Only in the IR, where it only makes sense to talk about operators, the additional rotations show up, transforming mesons and composite gauge invariants into each other. Of the invariants like V^2S^2 only the part transforming like $\boxplus$ under the diagonal $SU(3)$ is mapped to a magnetic gauge singlet, the other components are composite gauge invariants.

Similarly one can perturb the G_2 theory along its $SU(3)$ flat direction. We get a self-duality for $SU(3)$ with 5 flavors. For the same reasons as in the F_4 case we expect to see

only the diagonal subgroup of the global $SU(5) \times SU(5)$ flavor rotations. This leads us to the following self-dual pair:

	$SU(3)$	$SU(3)_L$	$SU(3)_R$	$U(1)$	$U(1)_R$
Q	3	5	1	1	$\frac{2}{5}$
$\tilde{Q}$	$\overline{3}$	1	5	-1	$\frac{2}{5}$

$\updownarrow$

	$SU(3)$	$SU(5)_D$	$U(1)_R$
q	3	5	$\frac{1}{3}$
$\tilde{q}$	$\overline{3}$	5	$\frac{1}{3}$
M	1	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\frac{2}{3}$
B	1	$\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$	$\frac{4}{3}$

with: $W = M(q^3 + \tilde{q}^3) + Bq\tilde{q}$

This new self-duality can be generalized to SUSY QCD with arbitrary gauge group and $N_c + 2$ flavors. The models with even N_c satisfy our Δ condition ($\Delta = 2N_c + 4 - 2N_c = 4$, $\mu_{gauge} = 2N_c$) and the odd values can be obtained by higgsing. Only the diagonal subgroup of flavor rotations is visible in the magnetic theory. The antisymmetric part of the meson and one baryon appear as elementary gauge singlet fields on the magnetic side, the symmetric part of the meson and the other baryon are composite fields.

There is another piece of evidence for the G_2 model: In addition to the *Spin* models of the two series in section 2, there is another self-dual model for $Spin(7)$:

electric:

	$Spin(7)$	$SU(6)$	$U(1)$	$U(1)_R$
Q	7	1	6	$\frac{2}{7}$
S	8	6	-1	$\frac{2}{7}$

magnetic:

	$Spin(7)$	$SU(6)$	$U(1)$	$U(1)_R$
q	7	1	6	$\frac{2}{7}$
s	8	6	-2	$\frac{2}{7}$
M_3	1	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$+4$	$\frac{6}{7}$
M_4	1	$\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$	-4	$\frac{6}{7}$

with: $W = M_4 s^2 q + M_3 s^4$.

mapping S^4 to M_4 and S^2Q to M_3.

This theory also s-confines after giving a mass to one vector. It is not a trivial consequence of the $Spin(8)$ models and triality, since triality is hidden in the magnetic $Spin(8)$ theories (it appears only in the infrared). After following this theory along a spinor flat direction we reproduce the G_2 theory. Following the vector flat direction we get $Spin(6) = SU(4)$ with 6 flavors and hence we again hit SUSY QCD with $N_c + 2$ flavors, which we allready showed to be self-dual.

4.2. Self-dual models with symmetric tensors

Even though self-duality created a lot of knowledge about SU theories with antisymmetric tensor representations, much less has been achieved in theories with symmetric tensors. Self-dualities with symmetric tensors could be very interesting, since these theories appear again in a different context: Pouliot's dualities [9] relate $Spin(7, 8, 10)$ groups with spinorial matter and SU groups with symmetric tensors. It would be very interesting to see if this can be generalized to higher $Spin$ groups, since there is no reason why 7, 8 and 10 should be special. If this is true, the many self-dual $Spin$ groups we presented should also have self-dual SU groups as duals (if A is dual to B and also self-dual, B has to be self-dual, too). Hence later ones would appear as natural starting points for more dualities a la Pouliot, which may lead to a deeper understanding of $N = 1$ dualities.

In this light it is very disappointing that, as one can check easily, there is no way to create a self-dual pair with the simple pattern we described in this paper, at least as long as it only involves one symmetric tensor and fundamentals and antifundamentals. Nevertheless we can create new self-dualities with SU groups, symmetric tensors and an additional superpotential by using some of our new self-dualities for groups that are known to also have a dual description in terms of a SU theory and applying the A dual B argument in the other direction. We will demonstrate this at the example of G_2 with 6 flavors. Besides the self-duality we presented, this is also dual to ([9])

	$SU(3)$	$SU(6)$	$U(1)_R$
q	$\overline{3}$	$\overline{6}$	$\frac{1}{3}$
q^0	$\overline{3}$	1	$\frac{2}{3}$
s	6	1	$\frac{2}{3}$
M	1	21	$\frac{2}{3}$

with $W = q^0q^0s + s^{N_f-3} + Mqqs$.

Now this has to be self-dual, too. In fact it is very easy to see that this is actually true, the dual being

	$SU(3)$	$SU(6)$	$U(1)_R$
Q	$\overline{3}$	$\overline{6}$	$\frac{1}{3}$
Q^0	$\overline{3}$	1	$\frac{2}{3}$
S	6	1	$\frac{2}{3}$
M	1	$\square\!\square$	$\frac{2}{3}$
B	1	$\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}$	1

with $W = Q^0Q^0S + S^3 + MQQS + BQ^3$.

One can get rid of the field M on both sides by giving it a mass. But we have to live with the other superpotential terms. We created, as advertised, a new self-dual pair of SU theories with symmetric tensor and tree level superpotential. A similar construction is possible in other cases where we allready know a dual.

5. Conclusion

We found a criterion that allows us to find candidates for self-duality. With this tool, we were able to create several new self-dual pairs, including some with exceptional gauge group. Even though for many of those theories this self-duality is all we have so far, we strongly suspect that there often is yet another dual description, also valid for more general matter content. Self-duality might be used as a tool in constructing those dualities.

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